Oct 19-9:46 AM

4.5 Linearizations and differentials

$y = f(x)$

$\boxed{L(x) = f'(a)(x-a) + f(a)}$ tan line

$y = m(x - x_1) + y_1$

Find the linearization of $f(x) = \sqrt{1+x}$ at x=0

Use the linearization to approximate $\sqrt{1.02}$

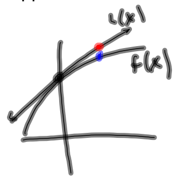
Use a calculator to determine the accuracy of the approximation.

$f'(x) = \frac{1}{2}(1+x)^{-\frac{1}{2}} \Big|_{x=0} = \frac{1}{2}$

$L(x) = \frac{1}{2}(x-0)+1 = \frac{1}{2}x+1$

$\sqrt{1.02} \approx \frac{1}{2}(.02)+1 = 1.01$

calc: $\sqrt{1.02} = 1.00995$



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Use Linearizations to approximate $\sqrt{123}$

$x=123$ $a=121$ easy & close to x

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$L(x) = f'(a)(x-a) + f(a)$

$= f'(121)(x-121) + f(121)$

$= \frac{1}{2\sqrt{121}}(x-121) + \sqrt{121}$

$= \frac{1}{2 \cdot 11}(x-121)+11$

$L(x) = \frac{1}{22}(x-121)+11$

$\sqrt{123} \approx \frac{1}{22}(123-121)+11$

$x=123$ $\frac{1}{22} \cdot 2 + 11$

$\frac{1}{11} + 11$

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Differentials

$\boxed{dy = f'(x)\,dx}$

$\frac{dy}{dx} = f'(x)$

Given $A = \pi r^2$ find the differential *dA* and evaluate *dA* for r=10

$\frac{dA}{dr} = 2\pi r$

$dA = 2\pi r \cdot dr$

$dr = \Delta r$

$dA \approx \Delta A$

What does the differential *dA* represent?

if $dr = .1$

$\Delta A \approx dA = 2\pi \cdot 10 \cdot (.1) = 2\pi$

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Percentage change is estimated by $\frac{df}{f(a)} * 100 = \frac{dy}{y} \cdot 100$

Ex 10. If the radius of the earth is estimated to be 3959 ± 0.1 what effect would the tolerance of 0.1 have on an estimate of the earth's surface area?

$S = 4\pi r^2$

$\Delta S \approx dS = 8\pi r \, dr$

$\Delta s \approx ds = 8\pi \cdot 3959 \cdot (0.1)$

% change:

$\frac{8\pi \cdot 3959 \, (0.1)}{4\pi \cdot 3959^2} \cdot 100\%$

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